THEORETICAL BASIS FOR MATHEMATICAL MULTIMEDIA SOFTWARE

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Abstract — *In this paper, we analyse the state-of-the-art of multimedia applications in the teaching of Mathematics. It will be studied in the light of the Theory of Conceptual Fields [1]Several works demonstrate that the educational softwares developed for the teaching of Mathematics tend to present a limited number of cases (Balacheff e Kaput, 1996; Schwartz, 1999; Hinostroza e Mellar, 2001). We believe that this is due to intrinsic limitations of the interfaces, which employ only graphical resources and direct manipulation. We shall discuss the main improvements in this area and present a model for the production of multimodal interfaces – those with multiple integrated forms of presentation, such as video and animation – oriented by a constructivistic proposal of cognitive development learning in this area.*

INTRODUCTION:

There is a plethora of educational software for the teaching of Mathematics available in the market, the majority strives to teach through successive repetitions of the solving of problems of the same type. At the end of this "marathon of exercices", the pupil feels that he has learned something. Such sensation can't stant modifications in the style of the questioning making it clear that, instead of learning the real contents, the student was simply trained to solve problem of a given pattern. According to Mayes [2], learning is the product of other activities. In this context, it is necessary to define and validate the models and methodologies that identify and evaluate the activities that cal help the student to learn, and translate these activities into educational software. We noticed the need of creating rich contexts, a wide repertory of situations, and the need of providing interfaces flexible enough to allow the emergence of multiple heuristics for the solving of problems in this conceptual field.

One of the characteristics of Mathematics is that its concepts are buildt on other concepts. The structure is not like a tree with many branches, but as a scaffold with many interconexions. Once the base of the scaffold is on a stable place it is not difficult to build it higher, but it is impossible to build one layer before the previous is finished. Due to this structure, it is almost always necessary that the teacher to impose some structure on the learning process. Our focus follows the Constructivistic Cognitive Theory.

THE LEARNING OF MATHEMATICS

The construtivism is rooted on the Piaget's works on the cognitive development of children and teenagers. His cognitive theory employs stages of development: starting with the senso-motor stage (which precedes speech) up to the stage where formal operations are performed. In this stage the child is capable of hypothetical-deductive reasoning [3]. Robert Shwarzenberger [4]observed that it is a reasonable hypothesis to suppose that the various cognitive mechanisms which control the childrens' individual learning are not different, qualitatively speaking, from the students'.

The Constructivistic Theory separaters training from teaching. In the first, the student is trained to produce an appropriate answer. The teacher imposes what is right or wrong, and the students absorb these facts, or not (which usually happens). The Construtivism asserts that teaching aims to produce autonomous understanding, and this shoud be the result of the student's mental process.

An important theoretical reference for the learning of Mathematic is Vergnaud's Theory of Conceptual Fields [1], according to which a concept is defined from three instances: its invariant properties, the systems of representation, and the instructions of use. Therefore, learning a mathematical concept implies to command a set of properties that emerge from different situations, and are mediated by different systems of representation. To command a conceptual field means to be capable of solving problems in different situations in which a concept is inserted.

The concept, the second element analysed in the represention process, and which assumes, in this work, an important position, is defined by Vergnaud [1]as a "tripod of sets"- the first set, named "S", containing the situations that convey meaning to the concept; the second, "I", containing the invariants on which the operationality of the schemes (meaning) is based; and "Y", the set containing the systems of representation, languages, which allow to represent symbolically the concept "Y" (significant). The situations ae aspects related to the deeper structure of the problems, and do not simply correspond to the contexts of the problems but to the relations between the quantities (numbers) which should occur to the subjects at the moment when they are organizing their actions for the tackling of the problems. The second element of the tripod is the set of the invariants

When interacting with real world, the individual will set in motion the knowlegde he has at that moment. The

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knowledge that emerges simultaneous to the actions are the invariant operatives, defined as the subject's knowledge that is subjacent to his behaviours, and that are, then, an integral part of his schemes of action. The articulated knowledge is, then, fragments of concepts which are employed in situations. The third component of the concept is that of symbolization, or significant. It is the way the individual is capable of expressing his thoughts. According to Vergnaud [1], there are at least six different types of situations involving these concepts; which must be completely mastered in order to be a real understanding of the conceptual field. They are:

TABLE 1
ADDITIVE SITUATIONS

I. Composition of measures - Ex: John has 12 shuttlecocks, and Peter 17. How many shuttlecocks they have together? There are two quantities given simultaneously, which the individual must start with to find a third quantity.	II. Transformation of measure - Ex.: Mary had 23 bombons. At the end of the day, she noticed that she had only 17 left. How many sweets bombons Mary ate on that day? The inicial and final qiuantities are known, but what is asked is the value of the variation between the first and the second moments.
III. Comparison of mesures - Ex.: I have 16 books, you have 43. How many books do you have more than I do? Once more, the quantities are known and simultaneous, aiming to compare the difference between the two (relation).	IV. Composition of transformations - Ex.: In the first match I scored 12 points, in a second match I scored 13 points. How many points did I score in all? The initial, intermediary, and final quantities are unknown. Only the transformations that take place during a span of time are known.
V. Composition of relations - Ex.: Mary is 3 years the senior of Antony. Marc is 4 years the senior of Mary. By how many years is Marc older than Antony? In this composition between two simultaneous relations, the elements are unknown but the relations are given.	VI. Transformation of relations - Ex.: *Mary had 3 toys more than John. She was given 4 more toys. Now, how many toys she has more than John?* None of the quantities are given, however, the existing relations and its alterations are known.

EDUCATIONAL SOFTWARE AND MULTIMEDIA IN THE TEACHING OF MATHEMATICS

The bibliography have been criticizing the quality of the educational software that seem not to correspond to the expectations of the teaching professionals [5]. The epistemological impact of the application of this new technology to the processes of learning and teaching is much smaller than the expected [6]. These results go against the evidence that the use of concrete materials – tangible interfaces – favours the learning of Mathematics [7]. Our hypothesis is that one of the reasons for this inefficiency is the fact that the process of designing considers certain aspects of the learning process in a superficial way, privileging aspects traditionally focused on the design of interfaces.

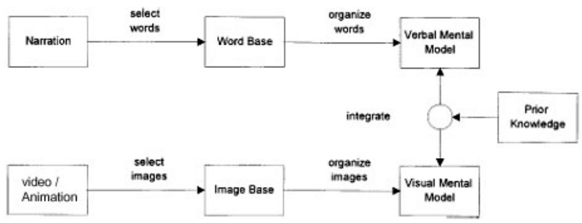
The process of educational software creation and evaluation demands the identification, or deduction of the knowledge that will probably emerge during the users' interactions with the interface, which take place in a non systematic way [8]. Having a systematic vision of the relation between the use of the interface and the actual learning implies in the use of a theoretical model of analysis that can do the modelling of the process of organization of the interaction with the interface in an understandable fashion. This modelling should happen in terms of theoretical elements that reveal the progressive adaptation of the users to the interface, and the knowledge that springs forth during its use.

The evidence gathered in our experience in the teaching of Methodology of the Teaching of Mathematics, point out that it is much simpler to negociate the meaning of mathematical relations through a dialogue than through the creation of an educational software interface. Anyway, educational interfaces are always limited when seen from the point of view of the functionalities related to the negotiation of meaning. The feedback provided are always restrictive, and in a few cases adapted to the situation experienced by the users.

Our aim is to promote a constructivistic learning, where the pupil is actively involved in the cognitive process, building significant representations and mental models.

In a constructivistic paradigm, the learning happens when the students' needs correspond to the visual and verbal representations of the memory of work at the same time. Researchers in the University of California [16] propose a multimedia learning model referenced by five principles, and by the multimedia learning cognitive theory: 1) Principle of multiple representation: it is better to present an explanation in words and graphics than only in words. This theory is corroborated by tests performed by Mayer and Anderson (1991) [17]; 2) Principle of contiguity: it is better to present words and corresponding images simultaneously than separately, offering thus a multimedia explanation; 3) Principle of coherence: multimedia explanations are better comprehended when they include only the needed information; 4) Principle of modality: it is better to present words in the form of a narration than as texts on the screen; 5) Principle of redundancy: it is better to present only animation with narration than animation, narration e and written text. The cognitive construction depends on the student's cognitive processing during the learning. The figure below shows the scheme for the Cognitive Theory of Multimedia Learning

FIGURE 1
SCHEME FOR THE COGNITIVE THEORY OF MULTIMEDIA.LEARNING



Words are represented as narrations, and images as animations or video. A cognitive apprentice will pay attention to relevant portions of the narrative (indicated by the arrow *selected words*) and will keep these words inside the verbal working memory (indicated by the box *word base*) analogously, he will pay attention to the relevant parts of the animation (indicated by the arrow *select images*) and will keep this images in the visual working memory (indicated by the box *image base*). Then , a active student will build conections mentally that organize words and images (indicated by the upper arrow that symbilizes the verbal channel, and the lower one that represents the visual channel) in a cause-effect chain (indicated by the boxes *Verbal Mental Model and Visual Mental Model*). Finally, he will build conections between the verbal and visual models, and his previous knowledge (indicated by the *integrate* structure). It is important to stress that these processes seem to take place interactively, instead of in predefined steps.

In order that a significant learning takes place, the student has to pass by each one of these cognitive processes. Selecting relevant words and images, organizing them in coherent verbal and visual representations, and integrating correspondin verbal and visual representations. All multimedia messages deliver information to the students, but not all are equally successful in promoting the understanding. The method we suggest was tested and approved by Richard E. Mayer *, Roxana Moreno [18]

ANIMATION

The theoretical foundation for the use of animation is the same for the use of static images [9]. Animations, just like static images, are considered as a subdivision of graphics in general. Pavio [10], in his dual theory of codification says that text and graphics are codified in two independent cognitive subsystems. Although keeping many similarities, animations have a series of properties, for instance, showing movement, which static images and graphics don't.

Animations have many functions. Among them we could cite: Attracting attention (when used to catch the attention); Motivating (when used as a feedback); Presenting contents (when originating from a concrete reference, and a visual context for ideas); Clarifying concepts (when propiciates a conceptual understanding even when not presenting new information). It is particularly important to catch the students' attention at the beginning of the lesson [19]. Animations can be used to attract the students' attention to key points. The clarifying function of the animation can offer an conceptual understanding even when it does not provide new information. Animations can help to exemplify abstract situations that, otherwise, would be hard to explain.

The concern in respecting preexistent patterns, in terms of the way the objetcs look in the animations, produces a positive effect in the understanding of the information presented. The coherence with other materials at the students' disposal is crucial since if the representation used in a textbook differs from the one used in the animation, this difference demands a greater cognitive effort from the student. Researches with concrete static images showed that abstract representations (schemes) are more easily remembered than concrete images [11]. On the other hand, if the goal of the instruction is a conceptual global recognition, the students will benefit from more realistic representations, such as videos.

According to Park [12], the greater the complexity of a concept, the greater the potential of understanding through the use of animation. He bases this argument on the fact that if the contents is very difficult of describing verbally, then animation can be employed effectively. [20] assert that animations help to create a mental picture of the system. When the differences between two very close concepts are not easily noticeable, then an animation can clarify the point.

VIDEO

The use of vídeo aiming at educational ends is not new. Beichner [13] reports some works on Physics performed in the 80s. Zollman [14] suggest the use of interactive vídeos as the base for laboratory tools for the modelling of complex events and qualitative analysis, among others. Other references are the work done by Marcuso and Webber [15], VideoPort (Pasço Scientific, 1999), and VideoGraph. The reasons for employing video in multimedia softwares, by its nature and the type of information it can convey, are the same for the use of animation. We can single out some advantages of the video. It allows the student to watch the real phenomenon instead of a simulation.

Research performed in laboratories for the teaching of Physics based in video show that both the students that worked with video, and the ones that worked with real experiments were able to obtain similar experimental data in the majority of experiments; in other situations, the students working with video could take advantage of the possibility of observing the phenomenon in slow motion or frame to frame. It is advisable that the pupil analyses and do the mathematical modelling starting with videos that show real situations (Rodrigues, 2001).

CONCLUSIONS

In this paper, we analysed the state-of-the-art of multimedia applications in the teaching of Mathematic. We noticed that the available bibliography in the field has a gap in dealing with the creation of interfaces that convey different contexts and situations. At the same time, it is important to offer interfaces flexible enough to allow the emergence of multiple heuristics in this conceptual field in order to produce an autonomous understanding stemming from the student's mental processes. The use of animation and videos associated to narrations propitiates greater assimilation of the contents presented, making them more efficient for the task of negociating themeaning of mathematical concepts. The animations can be used to build representations in order to facilitate the emergence of mental models. Presenting the problems with videos that picture the real world behind the mathematical concepts brings it closer to the students, and eliminates the possibility of double meanings in the interpretation of the real problem to be solved.

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