Analysis of Clipping in OFDM Symbols

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Abstract — *The purpose of this paper is to present the problems introduced in a Multicarrier Modulation (MCM) symbols by a nonlinear channel. The signal at the output of a multicarrier modulator has a very high peak to average power ratio and the amplifier can clip its peak values. In this paper, a simulation showing the interference of the clipping process in the performance of a multicarrier communication system will be presented with some commented results. The program used to simulate MCM systems has been developed using Matlab® platform and has the didactical objective of showing the principles involved on this issue.*

Index Terms — *Clip, Multicarrier Modulation, OFDM.*

INTRODUCTION

The advent of digital television and wireless local area network (WLAN) brought the necessity of transmission of digital data at high rates [1]. One technique that has being used to archive the required data rate is the Orthogonal Frequency Division Multiplexing (OFDM). The OFDM system is a multicarrier modulation where the frequencies of the sub-carriers are orthogonal. This technique is being widely used to transmit data at high rates because of its robustness to the multipath channel [1][2].

The OFDM signal is obtained by adding N orthogonal sub-carriers, each one modulated by a sequence of data. Thus, by the Central Limit Theorem [3], the OFDM symbol can be modeled as a Gaussian Process [4], where the amplitude of the signal has normal distribution. Because of it, the OFDM signal may have high peaks, which leads to a very high Peak to Average Power Ratio (PAPR). To transmit this signal, the power amplifier must have a wide linear operation region, which is difficult to archive. It means that the power amplifier may clip the high peaks of the OFDM signal, introducing a nonlinear interference in the transmitted signal. This paper will present a computer tool developed to analyze this nonlinear interference in the OFDM system, with some commented results.

CLIPPING OF AN OFDM SIGNAL

The OFDM system is based on parallel communication, where the data sequence is convert to N parallel sequences, which modulates N orthogonal sub-carriers. Figure 1 shows the block diagram of an OFDM modulator.

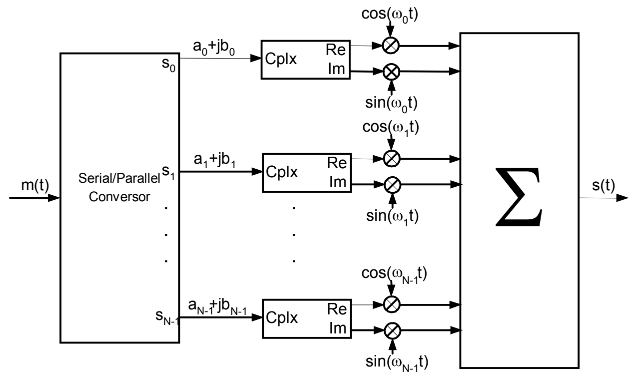

FIGURE 1
OFDM MODULATOR.

The data signal, $m(t)$, is the base band signal provided by any digital modulator, such as BPSK (Binary Phase Shift Keying), QPSK (Quadrature Phase Shift Keying) or QAM (Quadrature Amplitude Modulation), and it can be a complex signal. The data signal is converted to N parallel symbols, ($s_0, s_1, s_2, ..., s_{N-1}$), and the real component of each signal is modulated by a cosine function, while the imaginary component is modulated by a sine function. The angular frequencies of each sub-carrier are chosen to be orthogonal in order to satisfy (1).

$$\int_0^{Ts} \cos(\omega_i t)\cdot\cos(\omega_j t)dt = 0 \quad ; i \neq j \tag{1}$$

Where T is the OFDM symbol time.

If the signal $m(t)$ is a random sequence, the resulting OFDM signal, $s(t)$, can be modeled as a sum of N random variables, as shown by (2).

$$s(t) = \mathrm{Re}\left\{\sum_{i=0}^{N-1} s_i(t)e^{-j\omega_i t}\right\} \tag{2}$$

The Central Limit Theorem states that the sum of independent identical distributed (iid) random variables can be modeled as a gaussian variable with the variance (σ_s^2) and mean value (μ_s) given by (3).

$$\begin{aligned}\sigma_s^{\,2} &= N\cdot\sigma^2 \\ \mu_s &= N\cdot\mu\end{aligned} \tag{3}$$

Where σ and μ are, respectively, the standard deviation and mean value of the iid random variables. Figure 2 shows the probability density function (PDF) of an OFDM signal, with 64 carriers, modulated using QPSK.

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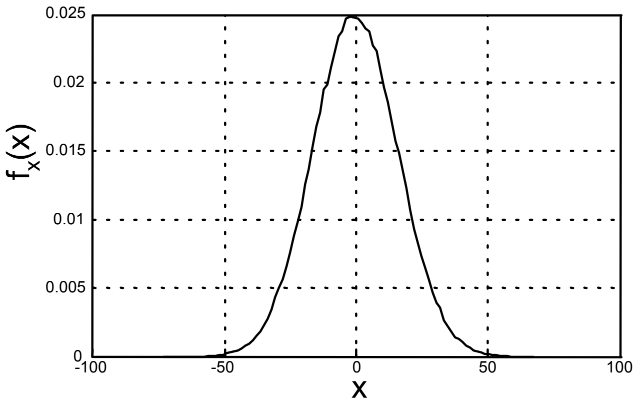

FIGURE 2
PDF OF AN OFDM SIGNAL.

It is possible to verify that the amplitude of an OFDM signal may present high values. To transmit this signal, it is necessary to apply it in an amplifier, with a very large linear operational region, but the amplifiers with this characteristic are expensive and inefficient [5]. Thus, the amplifiers usually are set for best performance and, because of it, its linear operation region is not as large as necessary to linearly amplify the OFDM signal. The high peaks of the OFDM signal may saturate the amplifier and this distortion can be modeled as a clipping effect. Figure 3 shows an OFDM signal clipped by a nonlinear power amplifier.

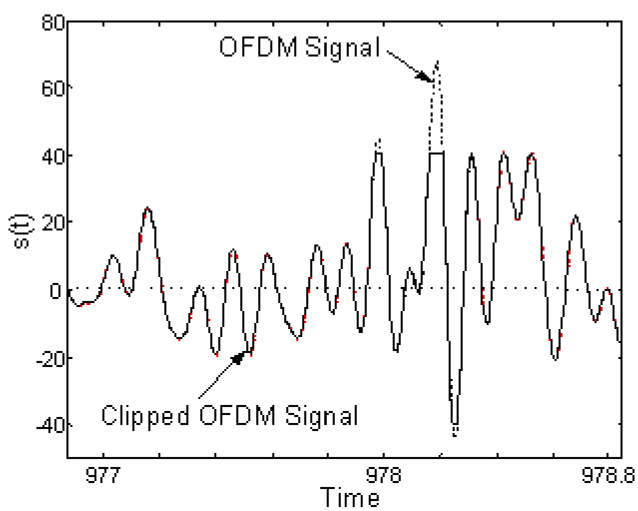

FIGURE 3
OFDM SIGNAL CLIPPED BY A NONLINEAR AMPLIFIER.

Once know why the clipping process occurs, it is important to define the influence of this interference in the system performance. In the OFDM system, the data symbols define the amplitude of each complex sub-carrier, as shown in (2). Because of this, the reception of the information is realized by detecting the amplitude of the sub-carriers in the frequency domain. Thus, the analysis of the clipping interference in the reception process should be done in the frequency domain instead of the time domain. If the number of sub-carrier (N) increases, the OFDM symbol time (T_s) also increases, as can be seen in (4).

$$T_s = N \cdot T \tag{4}$$

Where T is the symbol time of the incoming data signal, $m(t)$.

For large values of T_s, the clipped part of the OFDM signal can be approximated to an impulse, thus the clipping distortion can be modeled as an impulsive noise added to the transmitted signal, as shown in Figure 4.

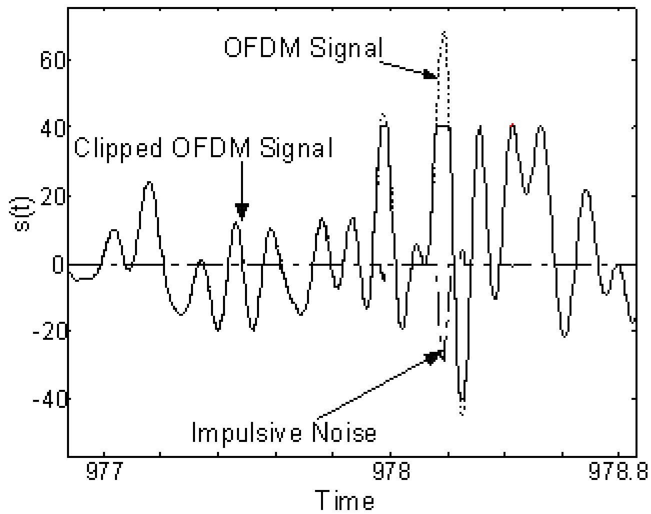

FIGURE 4
IMPULSIVE NOISE MODEL

It is known that the Fourier Transform of an impulse of amplitude α is defined by (5).

$$\mathscr{F}\{\alpha \cdot \delta(t)\} = \alpha \tag{5}$$

Equation (5) shows that the impulsive noise, in the frequency domain, represents a constant value which interferes in every sub-carrier. Thus, the impulsive noise does not destroy the information transmitted by a specific sub-carrier, but it introduces a distortion in every transmitted sub-carrier.

INFLUENCE OF THE CLIPPING DISTORTION

The reception of an OFDM signal can be implemented by a bank of correlators [6], as shown in Figure 5. The received signal, $s'(t)$, is multiplied by the N complexes sub-carriers and integrated in a OFDM symbol time. The estimated received symbols, ($s'_0, s'_1, s'_2, ..., s'_{N-1}$), are converted from parallel to serial format, generating the estimated data signal, $m'(t)$.

To analyze the effect of the clipping process, an OFDM system with 128 sub-carriers modulated with a QPSK scheme has been simulated. Figure 6 shows the transmitted data sequence, $m(t)$.

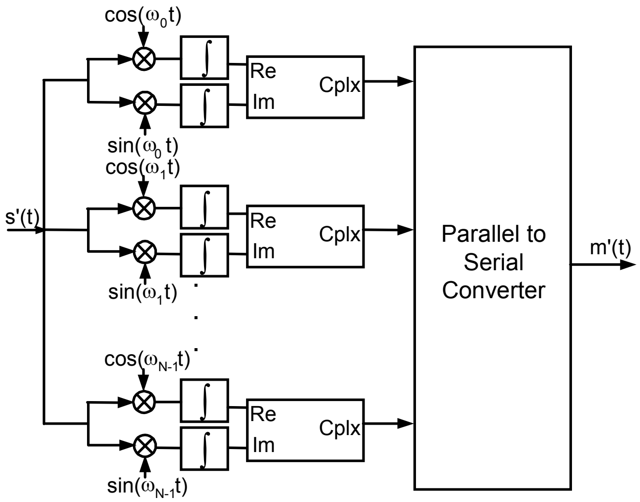

FIGURE 5
OFDM RECEIVER

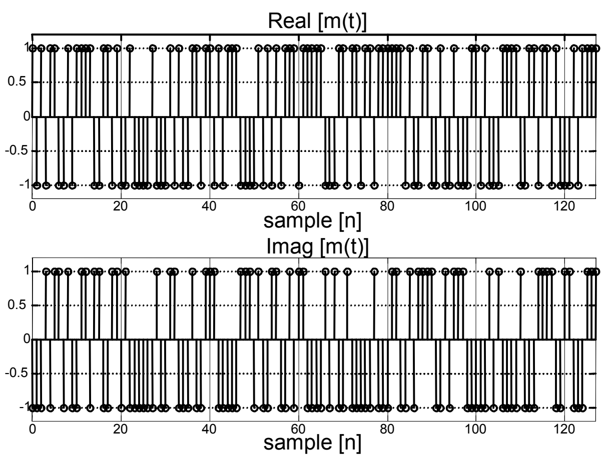

FIGURE 6
TRANSMITTED DATA SEQUENCE

The data sequence, presented in Figure 6, is applied to the OFDM modulator, generating the transmitted signal, $s(t)$, presented by Figure 7.

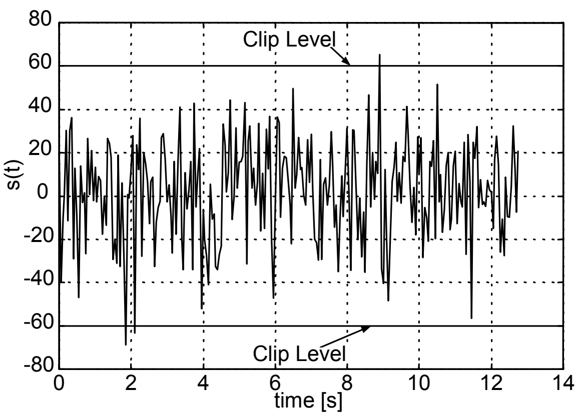

FIGURE 7
OFDM SIGNAL.

In this simulation, the clip level has been set to 60, which means that only the peaks above 60 and bellow –60 have been clipped. It is possible to observe that the introduced clipping distortion is not very high, in this case. Figure 8 shows the received data sequence, $m'(t)$.

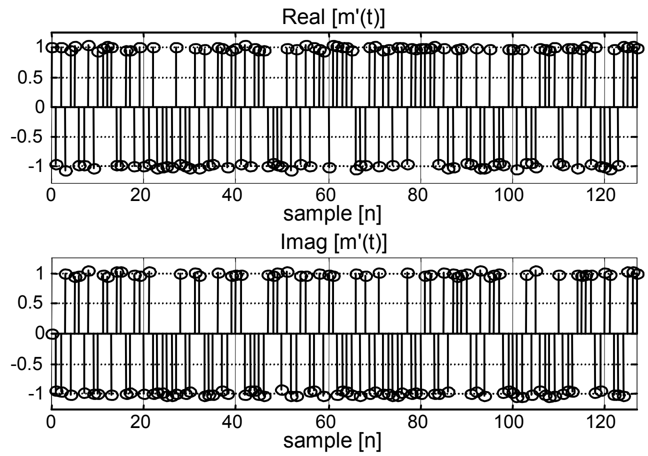

FIGURE 8
RECEIVED DATA SEQUENCE.

The clipping distortion introduces a variation in all transmitted symbols, decreasing the performance of the OFDM system.

In this simulation, the clipping occurred only in a few instants of time, because the clip level was set to a high value. In the next simulation, the clip level has been set to 30, which will introduce more clipping events to the transmitted signal, as can be seen in Figure 9.

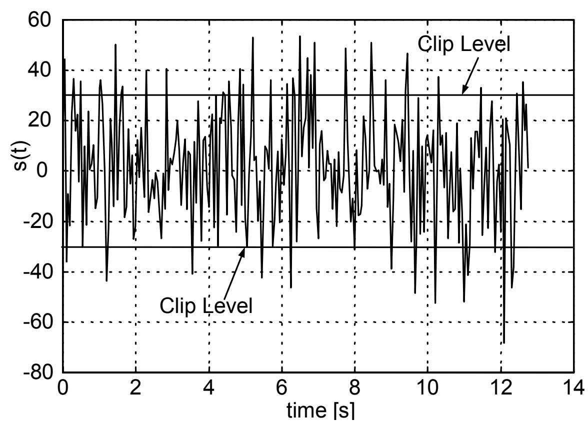

FIGURE 9
OFDM SIGNAL.

The distortion introduced by the clipping process in this simulation will perturb the transmitted data signal, reducing even more the performance of the system in a channel perturbed by AWGN (Additive White Gaussian Noise).

Figure 10 shows the received data sequence, perturbed by the clipping presented by Figure 9.

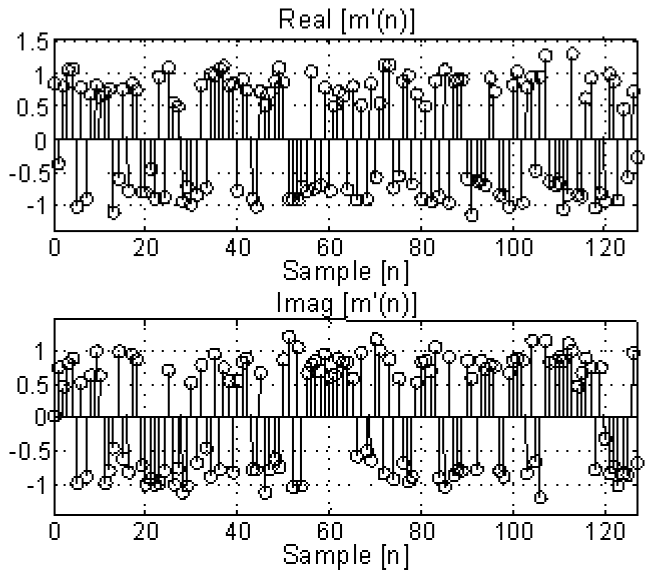

FIGURE 10
RECEIVED DATA SIGNAL.

As a quantitative parameter, the standard deviation can be used as a reference value for the clip level. In the first simulation, the clip level to the standard deviation rate was 2.66 (the standard deviation of the OFDM signal is 22.58). In the second simulation, the clip level to the standard deviation rate was 1.33. Thus, based on the simulations presented here, it is possible to observe that for ratio less than two, the interference of the clipping can be significant.

It is important to note that the clipping distortion does not introduce bit error, because the decision device can exactly approximate the received signal, *m'*(*t*), to the transmitted signal, *m*(*t*). However, the analysis realized in this paper, does not consider the AWGN of the channel. Thus, the clipping distortion will reduce the robustness of the system by increase the bit error rate (BER). The problems caused by the clipping can be minimized by applying a channel code for error control or by increasing the linear operation region of the power amplifier.

CONCLUSIONS

The OFDM signals can be modeled as a Gaussian Process and, thus, it may present very high amplitude values. The power amplifier used to transmit this signal usually presents a short linear region of operation due the power efficiency. Thus, the high peaks of the OFDM signal are clipped because the high values of this signal saturate the power amplifier. This distortion can be modeled as an impulsive noise that is added to the transmitted signal, and it interferes in every sub-carrier. The results presented in this paper, shows that the degradation in the system performance may be significant if the clip level is less than two times the standard deviation of the OFDM signal. It means that if the clip level is less than two times the standard deviation, the performance of the OFDM system in an AWGN channel can decrease significantly.

REFERENCES

[1] Nee, R, V and Prasad, R, *OFDM for Wireless Multimedia Communications*, Artech House, 2000.

[2] Bahai, A, R and Saltzberg, B, R, *Multicarrier Digital Communication: Theory and Applications of OFDM*, Norwell, MA: Kluwer Academic, 1999.

[3] Yates, R and Goodman, D, *Probability and Stochastic Processes*, Wiley, 1999.

[4] Bahai, A, R; Singh, M; Goldsmith, A, J and Satzberg, B, R, "A New Approach for Evaluating Clipping Distortion in Multicarrier Systems", *IEEE Journal on Selected Areas in Communications*, Vol. 20, No. 5, June 2002, pages 1037-1046.

[5] Lenk, J, D, *Optimizing Wireless/RF Circuits*, McGraw Hill, 1999.

[6] Bingham, J, a, C, "Multicarrier Modulation for Data Transmission: An Idea Whose Time Has Come", *IEE Communication Magazine*, 1990, pages 5-14.